Another Reason to Use the Hamilton Filter

March 21, 2021

Online Appendix

Appendix 1

Table 1.1 Cyclical Component of Output by Filter

	Hamilton		HP		Band-Pass	
Date	First Release	Final Release	First Release	Final Release	First Release	Final Release
1969:Q4	-1.19	-0.89	-2.13	-0.22	-0.76	-0.65
1970:Q1	-2.82	-3.02	-2.90	-1.11	-1.07	-1.54
1970:Q2	-4.52	-4.29	-3.23	-1.70	-1.58	-2.30
1970:Q3	-4.47	-3.67	-2.91	-1.52	-1.81	-2.66
1970:Q4	-5.77	-5.30	-3.61	-3.33	-1.91	-2.55
1971:Q1	-5.11	-4.66	-2.25	-1.39	-1.83	-2.18
1971:Q2	-4.46	-4.12	-1.33	-1.59	-1.29	-1.80
1971:Q3	-3.77	-3.87	-0.73	-1.52	-0.69	-1.53
1971:Q4	-2.21	-3.25	0.00	-2.04	-0.44	-1.22
1972:Q1	-0.39	-1.42	0.61	-0.97	-0.10	-0.61
1972:Q2	1.16	0.36	2.00	0.52	0.51	0.45
1972:Q3	2.30	0.41	2.35	0.71	1.14	1.79
1972:Q4	5.83	3.68	3.04	1.63	1.66	3.07
1973:Q1	4.89	2.75	3.28	3.35	1.97	3.94
1973:Q2	4.64	3.70	2.44	3.72	1.97	4.25
1973:Q3	5.82	3.05	1.90	2.50	1.68	4.07
1973:Q4	3.65	3.31	0.99	2.76	1.24	3.52
1974:Q1	1.31	0.37	-1.17	1.22	0.56	2.62
1974:Q2	-1.22	-1.59	-2.21	0.79	-0.48	1.36
1974:Q3	-3.13	-2.71	-3.13	-0.84	-1.28	-0.20
1974:Q4	-6.83	-4.93	-4.91	-1.91	-2.03	-1.78
1975:Q1	-11.30	-8.95	-6.63	-3.84	-3.09	-2.95
1975:Q2	-11.13	-8.73	-5.94	-3.85	-4.04	-3.37
1975:Q3	-8.49	-6.21	-2.56	-2.90	-3.69	-3.04
1975:Q4	-6.81	-6.70	-0.48	-2.33	-1.87	-2.29
1976:Q1	-3.51	-3.53	0.79	-0.89	-0.77	-1.61
1976:Q2	-1.89	-3.09	1.33	-0.97	-0.10	-1.32
1976:Q3	-1.12	-1.75	1.56	-1.25	0.02	-1.39
1976:Q4	1.55	-0.62	1.50	-1.37	-0.08	-1.57
1977:Q1	4.51	1.66	1.81	-1.04	-0.05	-1.53
1977:Q2	3.09	2.57	2.34	0.04	0.36	-1.18
1977:Q3	2.14	2.58	2.09	0.99	0.83	-0.61
1977:Q4	6.27	2.04	2.15	0.17	1.12	-0.01
1978:Q1	3.71	0.25	0.96	-0.32	0.86	0.55
1978:Q2	3.46	3.63	1.65	2.69	0.47	1.11
1978:Q3	4.55	4.14	1.54	2.93	0.53	1.72
1978:Q4	4.23	4.29	1.62	3.55	0.71	2.36
1979:Q1	2.33	3.22	0.85	3.04	0.93	2.81
1979:Q2	1.27	1.43	-0.76	2.50	0.60	2.85

	Hamilton		HP		Band-Pass	
Date	First Release	Final Release	First Release	Final Release	First Release	Final Release
1979:Q3	0.75	0.68	-0.74	2.63	-0.08	2.33
1979:Q4	0.54	1.44	-0.94	2.30	-0.57	1.41
1980:Q1	0.38	1.07	-1.18	2.06	-0.75	0.42
1980:Q2	-4.71	-5.79	-3.53	-0.56	-1.04	-0.25
1980:Q3	-4.61	-5.77	-3.34	-1.20	-1.65	-0.36
1980:Q4	-3.33	-4.74	-2.05	0.14	-1.66	0.02
1981:Q1	-3.33	-3.50	-1.15	1.56	-1.55	0.58
1981:Q2	-2.45	-4.42	-1.57	0.29	-1.11	0.89
1981:Q3	-4.40	-4.40	-1.84	0.94	-1.03	0.62
1981:Q4	-5.37	-5.58	-2.72	-0.71	-1.20	-0.30
1982:Q1	-6.23	-7.36	-3.29	-2.87	-1.77	-1.63
1982:Q2	-3.11	-4.57	-2.86	-3.05	-2.42	-2.96
1982:Q3	-4.52	-5.54	-2.35	-4.11	-2.42	-3.82
1982:Q4	-6.68	-8.03	-2.61	-4.80	-2.15	-3.93
1983:Q1	-5.86	-7.78	-1.41	-4.27	-1.63	-3.26
1983:Q2	-3.07	-3.75	0.00	-2.85	-1.20	-2.04
1983:Q3	-1.96	-3.67	1.51	-1.74	-0.28	-0.62
1983:Q4	-0.01	-0.58	1.84	-0.59	0.37	0.65
1984:Q1	2.88	3.16	2.91	0.39	0.85	1.55
1984:Q2	4.26	3.35	3.84	1.13	1.52	2.03
1984:Q3	5.17	5.01	3.14	1.10	1.88	2.14
1984:Q4	6.05	6.16	2.65	0.91	1.83	2.04
1985:Q1	5.50	5.44	1.91	0.86	1.58	1.85
1985:Q2	3.61	4.26	1.14	0.72	1.04	1.68
1985:Q3	3.83	4.40	0.99	1.22	0.65	1.52
1985:Q4	3.63	3.32	0.78	0.96	0.56	1.31
1986:Q1	0.71	2.32	0.43	0.89	0.39	0.99
1986:Q2	1.73	1.14	0.37	0.35	0.56	0.51
1986:Q3	1.80	1.21	0.07	0.34	0.20	-0.07
1986:Q4	1.26	0.80	-0.17	-0.08	-0.17	-0.62
1987:Q1	0.86	0.34	-0.02	-0.28	-0.48	-1.00
1987:Q2	1.44	0.55	-0.20	-0.12	-0.56	-1.14
1987:Q3	1.29	-0.16	-0.09	-0.15	-0.48	-1.07
1987:Q4	1.65	1.04	0.15	0.69	-0.32	-0.90
1988:Q1	0.94	0.67	-0.01	0.36	-0.29	-0.74
1988:Q2	3.38	1.43	0.34	0.85	-0.15	-0.62
1988:Q3	3.20	0.97	-0.03	0.64	-0.45	-0.50
1988:Q4	2.62	1.79	-0.32	1.21	-0.64	-0.33
1989:Q1	2.94	2.11	0.17	1.50	-0.55	-0.07
1989:Q2	2.83	1.68	-0.38	1.56	-0.32	0.24

	Hamilton		HP		Band-Pass	
Date	First Release	Final Release	First Release	Final Release	First Release	Final Release
1989:Q3	2.64	1.72	-0.44	1.63	-0.33	0.52
1989:Q4	1.15	0.16	-0.93	1.19	-0.50	0.70
1990:Q1	1.02	1.03	-0.97	1.66	-0.70	0.70
1990:Q2	0.14	0.03	-1.50	1.43	-1.07	0.49
1990:Q3	-0.61	-0.56	-1.60	0.91	-1.06	0.07
1990:Q4	-2.06	-2.79	-2.36	-0.57	-1.07	-0.50
1991:Q1	-3.65	-4.29	-2.91	-1.61	-1.26	-1.08
1991:Q2	-3.74	-4.05	-2.71	-1.41	-1.56	-1.47
1991:Q3	-3.68	-4.39	-2.26	-1.48	-1.70	-1.48
1991:Q4	-3.10	-4.21	-1.97	-1.73	-1.42	-1.05
1992:Q1	-2.87	-4.33	-1.45	-1.15	-1.03	-0.33
1992:Q2	-3.18	-3.49	-1.02	-0.70	-0.70	0.40
1992:Q3	-1.75	-2.38	-0.47	-0.36	-0.48	0.87
1992:Q4	-0.37	-0.52	0.29	0.01	-0.24	0.97
1993:Q1	0.53	-0.21	0.52	-0.51	0.05	0.80
1993:Q2	-0.30	-0.82	0.35	-0.65	0.16	0.60
1993:Q3	1.01	-0.47	0.65	-0.90	0.40	0.59
1993:Q4	2.80	0.87	1.35	-0.31	0.56	0.83
1994:Q1	2.36	0.42	1.41	-0.12	0.82	1.18
1994:Q2	3.12	0.82	1.54	0.42	0.88	1.41
1994:Q3	3.12	0.68	1.56	0.18	0.89	1.37
1994:Q4	2.91	0.75	1.78	0.48	0.92	1.03
1995:Q1	3.79	1.07	1.57	-0.03	0.94	0.49
1995:Q2	3.06	0.61	0.81	-0.61	0.73	-0.08
1995:Q3	3.37	0.83	1.05	-0.67	0.48	-0.55
1995:Q4	0.86	0.13	-0.01	-0.91	-0.15	-0.87
1996:Q1	1.26	0.10	0.12	-1.11	-0.33	-1.06
1996:Q2	1.01	0.56	0.33	-0.41	-0.40	-1.14
1996:Q3	0.67	0.97	0.30	-0.49	-0.33	-1.15
1996:Q4	1.45	0.78	0.68	-0.44	-0.28	-1.12
1997:Q1	1.66	1.09	1.02	-0.79	-0.32	-1.08
1997:Q2	1.71	2.49	0.69	-0.14	-0.27	-1.05
1997:Q3	1.79	2.58	0.88	0.10	-0.23	-1.03
1997:Q4	2.34	2.87	0.92	-0.05	-0.22	-0.98
1998:Q1	2.93	3.25	0.90	-0.05	-0.17	-0.86
1998:Q2	2.34	2.35	0.49	-0.12	0.01	-0.64
1998:Q3	3.14	3.02	0.43	0.14	-0.06	-0.36
1998:Q4	3.51	3.73	0.81	0.78	-0.07	-0.06
1999:Q1	3.19	3.92	0.88	0.78	0.06	0.25
1999:Q2	2.94	2.87	0.41	0.62	0.15	0.58
1999:Q3	2.67	3.07	0.17	1.02	0.00	0.98

	Hamilton		HP		Band-Pass	
Date	First Release	Final Release	First Release	Final Release	First Release	Final Release
1999:Q4	3.65	4.21	0.59	1.83	0.15	1.47
2000:Q1	3.22	3.40	0.98	1.34	0.57	1.95
2000:Q2	4.20	4.24	0.96	2.33	0.82	2.26
2000:Q3	4.25	3.12	0.51	1.67	0.82	2.24
2000:Q4	2.52	2.14	-0.29	1.53	0.54	1.84
2001:Q1	1.97	1.20	-0.77	0.50	0.17	1.18
2001:Q2	1.01	1.07	-1.44	0.36	-0.29	0.48
2001:Q3	-0.47	-0.93	-2.06	-0.77	-0.61	-0.07
2001:Q4	-2.67	-2.35	-2.47	-1.20	-0.97	-0.42
2002:Q1	-0.81	-1.38	-1.28	-1.02	-0.91	-0.64
2002:Q2	-3.09	-2.91	-1.50	-1.11	-0.79	-0.89
2002:Q3	-2.58	-2.44	-1.08	-1.35	-0.59	-1.20
2002:Q4	-2.32	-2.83	-1.09	-1.89	-0.48	-1.50
2003:Q1	-1.93	-2.24	-0.95	-2.03	-0.53	-1.65
2003:Q2	-0.98	-2.11	-0.79	-1.87	-0.63	-1.54
2003:Q3	0.58	0.04	0.44	-0.88	-0.38	-1.22
2003:Q4	1.03	0.88	0.77	-0.43	0.05	-0.85
2004:Q1	1.09	0.28	0.96	-0.59	0.46	-0.58
2004:Q2	1.79	0.78	0.88	-0.51	0.56	-0.48
2004:Q3	1.95	1.45	0.95	-0.23	0.50	-0.47
2004:Q4	2.68	2.24	0.88	0.11	0.40	-0.41
2005:Q1	2.96	2.64	0.86	0.58	0.38	-0.22
2005:Q2	2.12	2.19	0.90	0.44	0.41	0.07
2005:Q3	1.73	1.44	0.88	0.75	0.48	0.36
2005:Q4	1.74	1.26	0.40	0.83	0.45	0.58
2006:Q1	2.21	2.32	0.74	1.65	0.34	0.71
2006:Q2	1.32	1.53	0.64	1.41	0.27	0.78
2006:Q3	1.00	0.59	0.25	1.13	0.14	0.84
2006:Q4	1.22	0.55	0.34	1.59	0.06	0.92
2007:Q1	0.54	-0.25	-0.23	1.47	-0.11	1.05
2007:Q2	-0.19	0.01	-0.30	1.73	-0.44	1.26
2007:Q3	-0.03	-0.46	0.05	1.99	-0.40	1.53
2007:Q4	-0.23	-0.54	-0.22	2.36	-0.31	1.79
2008:Q1	-1.11	-2.45	-0.61	1.57	-0.43	1.81
2008:Q2	-1.54	-1.93	-0.64	1.89	-0.64	1.38
2008:Q3	-0.90	-2.59	-0.91	1.17	-0.75	0.39
2008:Q4	-2.84	-6.05	-2.02	-1.18	-1.08	-0.94
2009:Q1	-5.07	-7.21	-3.73	-2.47	-1.91	-2.19
2009:Q2	-7.40	-7.87	-3.72	-2.79	-2.76	-2.86
2009:Q3	-7.21	-8.16	-2.44	-2.61	-2.71	-2.72

	Hamilton		HP		Band-Pass	
Date	First Release	Final Release	First Release	Final Release	First Release	Final Release
2009:Q4	-6.46	-7.58	-1.19	-1.72	-2.15	-1.93
2010:Q1	-5.49	-6.42	-0.46	-1.56	-1.34	-0.95
2010:Q2	-5.10	-6.36	0.06	-0.89	-0.81	-0.23
2010:Q3	-4.45	-5.07	0.28	-0.43	-0.62	0.03
2010:Q4	-1.53	-1.99	0.86	-0.23	-0.50	-0.05
2011:Q1	-0.47	-1.87	0.94	-0.79	-0.33	-0.14
2011:Q2	-0.94	-1.49	0.86	-0.43	-0.21	-0.02
2011:Q3	0.05	-1.52	1.12	-0.83	-0.01	0.30
2011:Q4	0.37	-1.21	1.24	-0.08	0.16	0.64
2012:Q1	0.00	-0.42	1.34	0.28	0.31	0.80
2012:Q2	-0.51	-0.89	1.32	0.27	0.35	0.75
2012:Q3	-0.62	-1.56	1.24	-0.06	0.29	0.57
2012:Q4	-0.91	-1.77	0.93	-0.43	0.30	0.40
2013:Q1	0.63	-0.61	1.10	-0.04	0.28	0.28
2013:Q2	-0.17	-1.53	0.69	-0.44	0.10	0.18
2013:Q3	-0.41	-0.65	0.93	-0.19	0.07	0.04
2013:Q4	-0.17	-1.04	1.25	0.06	0.18	-0.10
2014:Q1	-1.08	-2.05	0.57	-0.78	0.08	-0.13
2014:Q2	0.37	-0.78	0.75	0.00	-0.14	0.02
2014:Q3	-0.17	0.19	1.03	0.63	-0.19	0.32
2014:Q4	0.89	0.50	1.17	0.61	-0.04	0.62
2015:Q1	0.70	0.19	0.50	0.81	-0.11	0.74
2015:Q2	-0.09	1.06	0.55	0.95	-0.35	0.60
2015:Q3	0.50	0.61	0.58	0.69	-0.41	0.28
2015:Q4	-0.10	-0.25	0.26	0.13	-0.51	-0.08
2016:Q1	1.28	0.91	0.00	0.03	-0.62	-0.35
2016:Q2	0.39	-0.39	-0.25	-0.10	-0.84	-0.50
2016:Q3	-0.41	-1.07	-0.03	-0.16	-0.86	-0.58
2016:Q4	-0.43	-0.56	0.01	-0.26	-0.77	-0.63
2017:Q1	-0.33	-0.93	-0.26	-0.31	-0.75	-0.67
2017:Q2	-1.22	-1.27	-0.19	-0.38	-0.81	-0.63
2017:Q3	-0.13	-0.67	0.07	-0.21	-0.71	-0.48
2017:Q4	0.33	0.14	0.14	0.04	-0.55	-0.24
2018:Q1	0.51	0.04	0.17	0.06	-0.39	0.00
2018:Q2	0.57	0.44	0.37	0.30	-0.36	0.17
2018:Q3	1.97	0.74	0.54	0.40	-0.19	0.23
2018:Q4	0.87	0.53	0.45	0.06	-0.05	0.20
2019:Q1	1.51	0.74	0.44	0.20	0.03	0.13
2019:Q2	1.88	0.73	0.05	0.08	-0.09	0.04
2019:Q3	1.27	0.44	-0.08	-0.02	-0.07	-0.03
2019:Q4	0.12	0.12	-0.12	-0.12	-0.06	-0.06

Figure 1: Hamilton Filter: First Release vs Final Release by Series

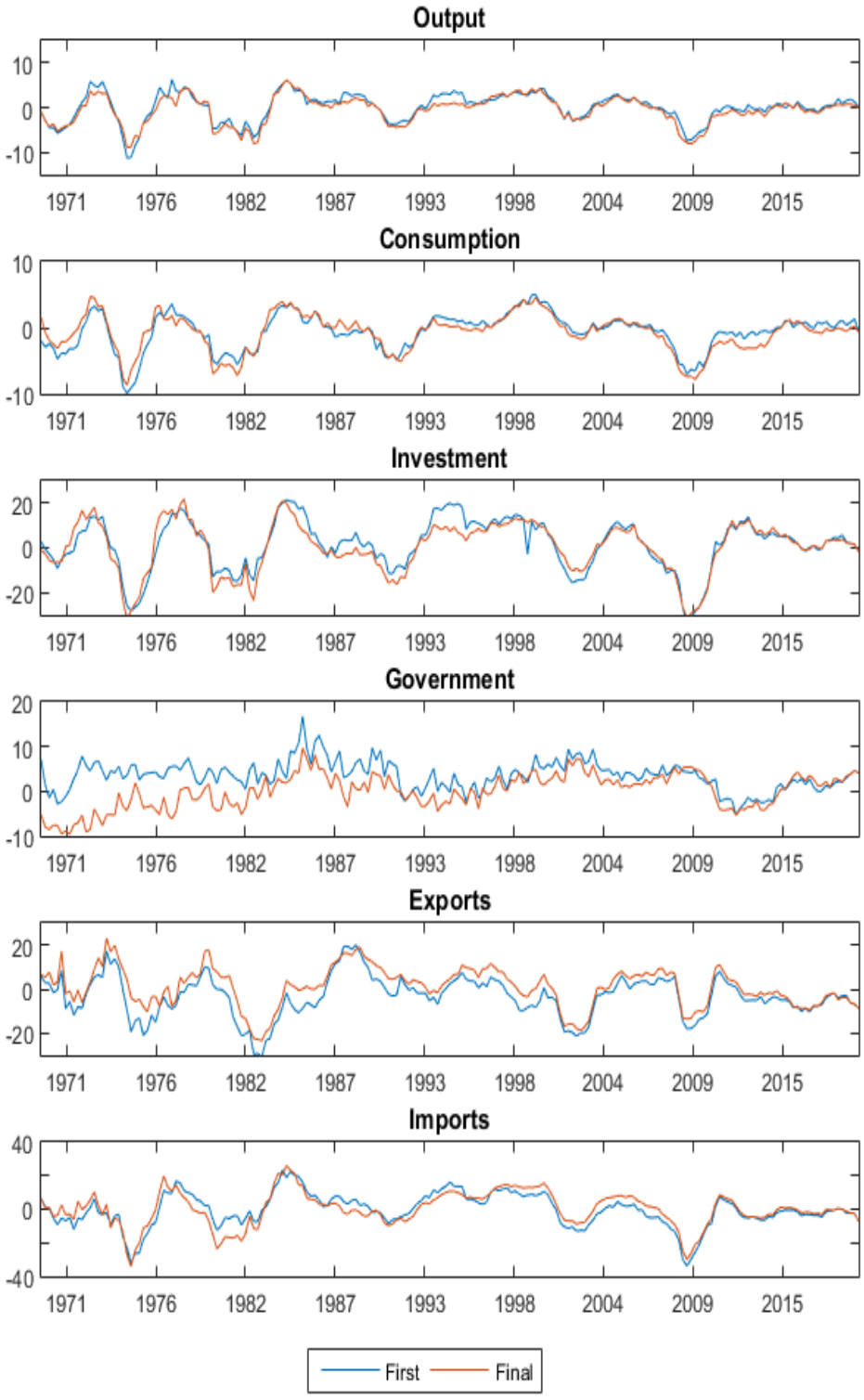


Figure 2: HP Filter ($\lambda = 1600$): First Release vs Final Release by Series

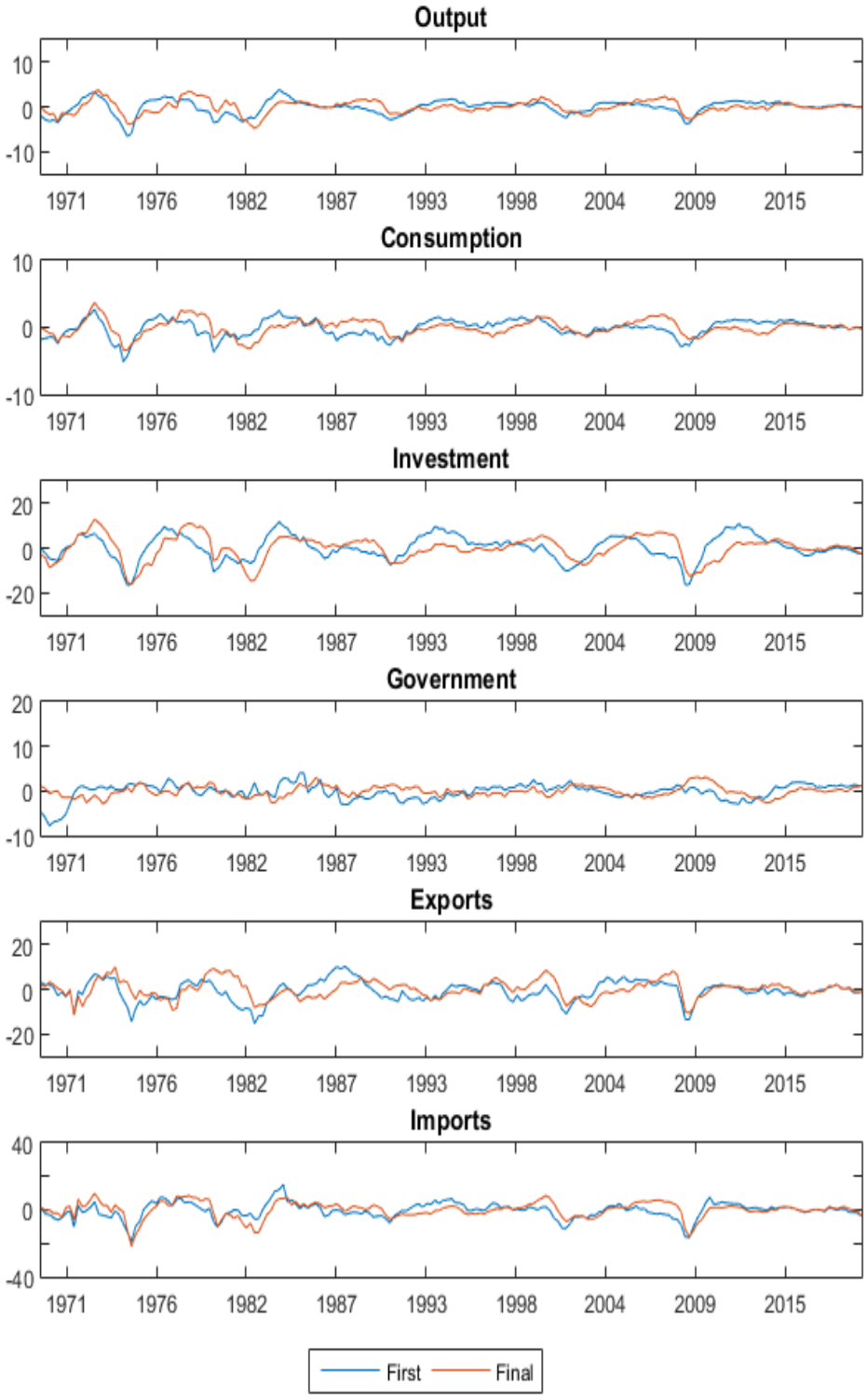


Figure 3: Band Pass Filter: First Release vs Final Release by Series

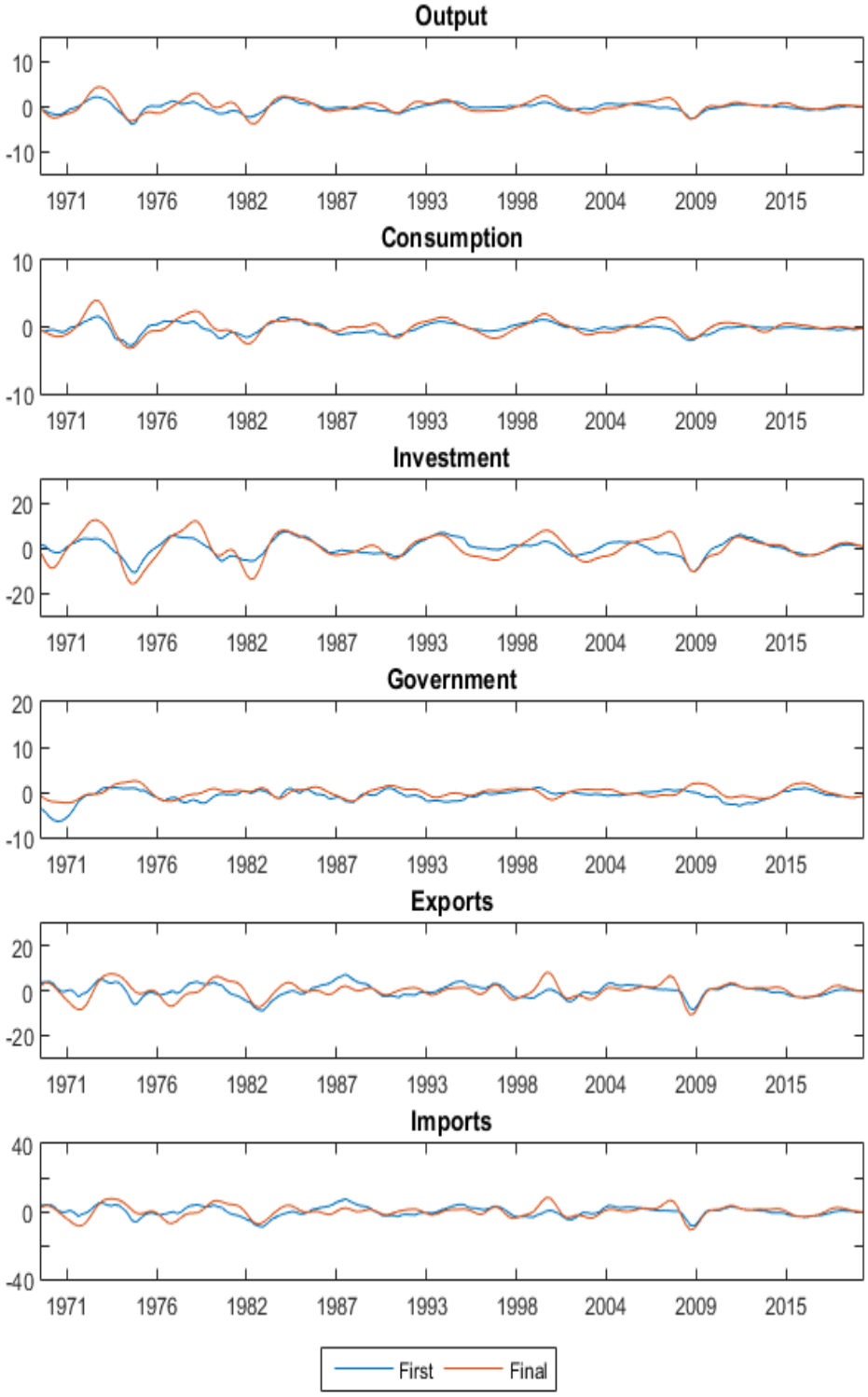
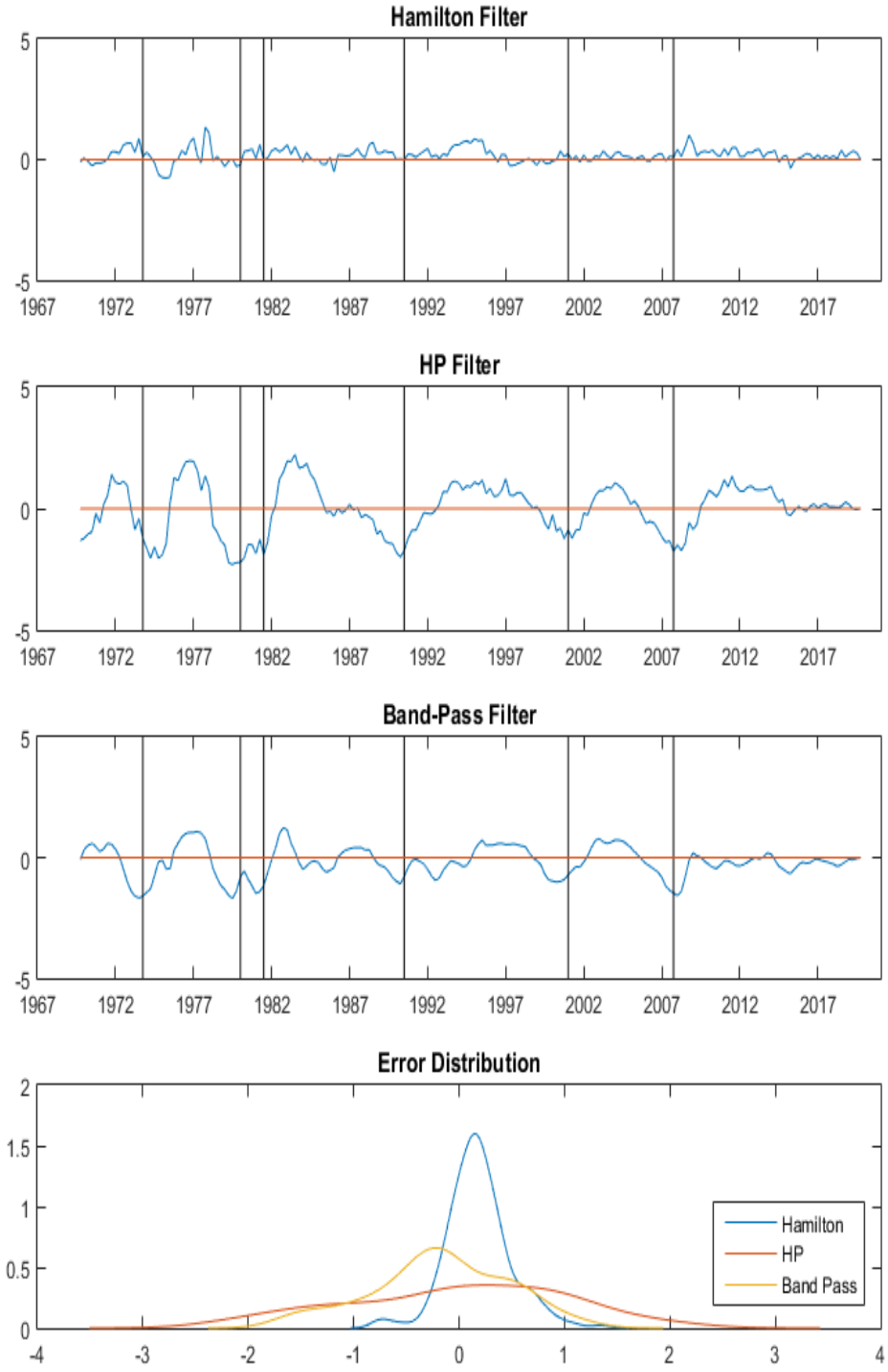


Figure 4: Error Output Series by Filter type



Appendix 2

Filters and Seasonal Adjustment Method

In this section, I apply each filter to seasonally adjusted and seasonally unadjusted macroeconomic time series and I compare the properties of the filtered series.

The Bureau of Economic Analysis of the Department of Commerce (BEA) collects and publishes series of seasonally unadjusted real GDP and it's major components since 2002. To construct a longer real seasonally unadjusted series, I use the seasonally unadjusted nominal quantity series for GDP and most of its major components (start period 1947), and I divide each of the nominal quantity series by the seasonally unadjusted Consumer Price Index. This is the same procedure used in Barsky and Miron (1989).

Table 2.1 shows the standard deviation of the seasonally adjusted and seasonally unadjusted cyclical components by filter type. A very interesting fact is that for most of the series considered, the standard deviation of the cyclical component is greater when the series is filtered through the Hamilton filter. Note also that when I apply the Hamilton filter or the Band-Pass filter, the standard deviation of the seasonally adjusted cyclical component is similar to the standard deviation of the seasonally unadjusted cyclical component. Not surprisingly, when the series is filtered through HP filter, the standard deviation changes considerably between seasonally adjusted and seasonally unadjusted.

Table 2.2 shows the standard deviation of the cyclical component for the same macroeconomics series extracted by applying the Hamilton filter, the HP filter (for values of $\lambda = \{1, 10, 100, 400, 1600\}$), the Band-Pass filter of Christiano and Fitzgerald (2003) and the Optimal Band-Pass filter of Baxter and King (1999) (for truncation values $K = \{2, 4, 8, 12, 16, 20, 24\}$). Throughout the table, standard deviations are computed for the time period associated with the shortest filtered time series (i.e, Optimal Band-Pass filter with truncation $K = 24$), so differences

in standard deviations are not due to differences in the sample period.[1]

The main conclusion remains. For most of the series analyzed, the Hamilton cyclical component is more volatile. Regardless of the value of λ used in the HP filter, the standard deviation changes considerably between seasonally adjusted and seasonally unadjusted. For the Optimal Band-Pass filter, the standard deviation is similar between seasonally adjusted and seasonally unadjusted when the truncation parameter $K \geq 4$. The standard deviation of the Optimal Band-Pass filter converges to the standard deviation of the Band-Pass Filter of Christiano and Fitzgerald for $K \geq 12$.

Table 2.3 shows the correlation between seasonally adjusted and seasonally unadjusted series by filter type. When the series are filtered through the Hamilton filter or the Band-Pass filter, the correlation coefficient is high. In the case of the Hamilton filter, in 24 of the 32 series analyzed, the correlation coefficient exceeds 0.9, while for the Band-Pass filter, in 26 of the 32 series analyzed, the correlation coefficient exceeds 0.9. In contrast, for the HP filter, the correlation coefficient exceeds 0.9 in 8 series.

Table 2.4 shows the autocorrelations for the seasonally adjusted and seasonally unadjusted cyclical components by filter type. The autocorrelation "match" between seasonally adjusted and seasonally adjusted is strikingly good when the series are filtered through the Hamilton filter and the Band-Pass filter. Instead, when the series are filtered through the HP filter, seasonally adjusted and seasonally unadjusted differ considerably. In general, the autocorrelations decline more smoothly when the series are filtered through the Hamilton filter.

In 2018, the BEA made changes to the method for seasonal adjustment of the National Income and Product Accounts (NIPA) data. Wright (2018) finds some indication of residual seasonality in the seasonally adjusted data as published before this update and the evidence

[1]Recall that in the Optimal Band-Pass filter with truncation parameter K, we lose the first and last K observations.

for residual seasonality is weaker in the seasonally adjusted data after the update. In addition, Wright (2018) proposes a new seasonal adjustment method, which consists of using TRAMO-SEATS, which is based on a seasonal ARIMA model that has the property of avoiding residual seasonality.

Table 2.5 shows the standard deviation of the cyclical component for NIPA data extracted by applying the Hamilton Filter, HP filter and Band Pass filter to (1) seasonally unadjusted series, (2) seasonally adjusted series before update (pre-revision official) , (3) seasonally adjusted series after the update (post-revision official) and (4) seasonally adjusted series through Wright's method (direct). For both, the Hamilton filter and the Band Pass filter the standard deviation is similar across different seasonal adjustment methods. For the HP filter, once the series has been seasonally adjusted, the standard deviation is also similar between different types of seasonally adjustment methods. The conclusion is that Hamilton and Band-Pass filter are robust to the seasonal adjustment method.

Table 2.1 Standard Deviation of the Cyclical Component by Filter

	Hamilton		HP		Band-Pass	
	Seasonally		Seasonally		Seasonally	
Series	Unadjusted	Adjusted	Unadjusted	Adjusted	Unadjusted	Adjusted
GDP	3.72	3.27	2.65	1.56	1.71	1.49
Consumption	3.24	2.76	3.19	1.24	1.40	1.17
Goods	4.47	4.46	6.65	2.01	1.98	1.86
Durable goods	10.19	9.87	8.66	4.76	4.68	4.37
Nondurable goods	3.17	2.59	6.26	1.13	1.39	1.03
Services	2.68	1.72	1.37	0.75	1.15	0.70
Investment	13.4	12.78	7.83	6.93	7.10	6.75
Fixed investment	10.16	9.71	6.75	4.71	4.77	4.64
Non-Residential	9.27	9.47	5.31	4.54	4.32	4.41
Structures	13.86	11.55	8.68	5.96	6.14	5.54
Equipment	11.52	12.61	6.78	6.09	5.52	5.95
Intellectual Property	6.75	6.49	3.23	2.90	2.77	2.84
Residential	21.11	18.99	15.40	9.49	9.54	9.11
Exports	13.25	10.58	6.69	5.07	6.00	4.59
Exports goods	15.51	12.05	7.97	5.87	7.01	5.23
Exports services	9.83	9.87	6.06	5.00	4.07	4.20
Imports	10.85	9.51	6.34	4.90	5.50	4.55
Imports goods	12.91	11.12	7.08	5.82	6.53	5.40
Imports services	8.72	9.65	8.88	4.19	3.37	3.43
Government	6.64	6.99	3.34	3.25	2.68	2.92
Federal	10.29	11.32	5.04	5.33	4.50	4.87
Defense	11.67	13.03	5.95	6.50	5.63	6.04
Non-Defense	13.65	10.63	9.75	5.03	5.64	4.66
State and local	4.12	3.65	2.93	1.49	1.37	1.39
Unemployment Rate	1.42	1.42	0.83	0.76	0.71	0.71
Employment	2.32	2.29	1.41	1.02	0.95	0.95
Average Hours	0.79	0.72	0.72	0.41	0.42	0.40
Labor Force	1.34	2.33	0.81	0.41	0.36	0.35
Price Level	3.96	3.94	1.27	1.25	1.22	1.22
Nominal Wage	1.61	1.53	0.65	0.60	0.32	0.31
Real Wage	2.59	2.55	1.15	1.07	1.11	1.10
Nominal Money Stock	6.36	6.19	2.59	2.52	2.09	2.09

The sample period is 1950:1-2019:2 except for Unemployment Rate and Labor Force (1951:1-2019:2), Nominal Interest Rate and Real Interest Rate (1957:3-2019:2), Nominal Monetary Base and Nominal Monetary Stock (1962:1-2019:2) and Average Hours, Nominal Wage and Real Wage (1967:1-2019:2).

Table 2.2 Standard Deviation of the Cyclical Component by Filter

	Hamilton		HP										Band-Pass	
			$\lambda = 1600$		$\lambda = 400$		$\lambda = 100$		$\lambda = 10$		$\lambda = 1$			
	SU	SA	SU	SA	SU	SA	SU	SA	SU	SA	SU	SA	SU	SA
GDP	3.8	3.3	2.7	1.6	2.4	1.3	2.2	1.0	2.0	0.6	1.8	0.3	1.7	1.5
Consumption	3.3	2.9	3.1	1.2	3.0	1.0	2.9	0.7	2.8	0.4	2.5	0.3	1.4	1.2
Goods	4.6	4.6	6.7	2.0	6.5	1.6	6.4	1.2	6.3	0.8	5.6	0.5	2.0	1.9
Durable	9.9	9.7	8.5	4.5	8.0	3.6	7.7	2.9	7.3	2.0	6.5	1.3	4.4	4.1
Nondurable	3.3	2.7	6.3	1.2	6.2	0.9	6.2	0.7	6.0	0.5	5.4	0.3	1.4	1.1
Services	2.8	1.8	1.4	0.7	1.1	0.6	0.9	0.4	0.7	0.3	0.6	0.2	1.2	0.7
Investment	13.6	13.3	7.5	6.9	6.3	5.7	5.2	4.5	3.8	2.9	2.6	1.7	6.8	6.7
Fixed investment	10.5	10.1	6.8	4.8	5.9	3.7	5.3	2.6	4.7	1.4	3.9	0.7	4.8	4.7
Non-Residential	9.6	9.8	5.4	4.7	4.5	3.7	3.7	2.6	2.9	1.4	2.5	0.7	4.4	4.5
Structures	14.3	12.0	8.9	6.1	7.5	4.8	6.6	3.5	5.7	1.8	4.6	1.00	6.1	5.6
Equipment	12.0	12.9	6.9	6.3	5.8	5.0	4.9	3.7	4.1	2.1	3.4	1.1	5.6	6.1
IP	6.8	6.5	3.0	2.6	2.4	2.0	2.0	1.5	1.5	0.8	1.1	0.4	2.6	2.7
Residential	21.5	19.4	15.8	9.7	14.1	7.3	13.0	5.2	12.0	2.9	9.8	1.5	9.8	9.3
Exports	12.6	10.0	6.3	4.8	5.4	4.0	4.5	3.3	3.4	2.3	2.8	1.6	5.2	4.0
Exports goods	15.0	11.7	7.7	5.7	6.6	4.8	5.5	3.9	4.3	2.7	3.5	2.0	6.4	4.8
Exports services	9.4	9.1	5.9	4.6	5.5	4.1	5.2	3.7	4.7	3.0	3.8	2.3	3.6	3.4
Imports	11.0	10.0	6.3	4.9	5.6	4.0	4.9	3.2	3.9	2.3	2.8	1.5	5.3	4.5
Imports goods	13.1	11.6	6.9	5.7	6.1	4.8	5.3	3.9	3.9	2.8	2.7	1.9	6.3	5.3
Imports services	7.8	8.8	8.8	3.8	8.6	3.2	8.5	2.8	8.2	2.2	6.7	1.7	2.9	3.1
Government	5.3	5.1	2.6	2.2	2.2	1.6	1.9	1.0	1.7	0.6	1.5	0.4	1.9	2.0
Federal	8.2	8.3	3.5	3.5	2.8	2.5	2.1	1.6	1.7	1.0	1.4	0.8	3.2	3.3
Defense	9.0	9.6	3.8	4.1	2.8	2.8	1.9	1.8	1.2	1.1	1.0	0.8	3.9	4.0
Non-Defense	11.6	8.9	9.0	4.4	8.7	4.0	8.4	3.5	8.0	2.8	6.6	2.2	4.3	3.7
State and local	4.0	3.3	2.9	1.3	2.7	1.0	2.6	0.8	2.6	0.5	2.3	0.3	1.2	1.2
Unemployment	1.4	1.4	0.9	0.8	0.7	0.6	0.6	0.5	0.4	0.3	0.3	0.1	0.7	0.7
Employment	2.4	2.4	1.5	1.1	1.3	0.8	1.2	0.6	1.0	0.3	0.8	0.2	1.0	1.0
Average Hours	0.8	0.8	0.7	0.4	0.7	0.4	0.7	0.3	0.6	0.2	0.5	0.1	0.4	0.4
Labor Force	1.3	2.1	0.8	0.4	0.8	0.3	0.8	0.3	0.7	0.2	0.6	0.1	0.4	0.4
Price Level	3.7	3.7	1.2	1.2	0.9	0.9	0.6	0.6	0.3	0.3	0.2	0.2	1.1	1.1
Nominal Wage	1.7	1.6	0.7	0.6	0.4	0.4	0.3	0.2	0.3	0.2	0.2	0.1	0.3	0.3
Real Wage	2.7	2.6	1.2	1.2	1.0	0.9	0.7	0.6	0.5	0.3	0.4	0.2	1.2	1.2
Money Stock	6.8	6.6	2.8	2.7	1.9	1.8	1.4	1.2	0.9	0.7	0.6	0.4	2.2	2.2

	Optimal Band-Pass													
	$K = 2$		$K = 4$		$K = 8$		$K = 12$		$K = 16$		$K = 20$		$K = 24$	
	SU	SA	SU	SA	SU	SA	SU	SA	SU	SA	SU	SA	SU	SA
GDP	0.5	0.1	0.9	0.8	1.4	1.2	1.7	1.5	1.7	1.4	1.7	1.4	1.7	1.4
Consumption	0.6	0.1	0.8	0.6	1.1	0.9	1.4	1.2	1.4	1.2	1.3	1.1	1.3	1.1
Goods	1.5	0.2	1.5	0.9	1.6	1.5	2.0	1.9	1.9	1.9	1.9	1.8	1.9	1.8
Durable	1.6	0.4	2.6	2.1	3.5	3.3	4.4	4.1	4.3	4.0	4.2	3.9	4.2	3.9
Nondurable	1.5	0.1	1.4	0.5	1.1	0.9	1.4	1.1	1.3	1.1	1.3	1.0	1.4	1.0
Services	0.2	0.1	0.5	0.3	0.8	0.5	1.1	0.7	1.1	0.7	1.1	0.7	1.1	0.7
Investment	0.9	0.6	3.5	3.4	5.5	5.4	6.6	6.4	6.5	6.2	6.5	6.3	6.6	6.3
Fixed investment	1.2	0.3	2.1	2.0	3.4	3.4	4.7	4.6	4.5	4.4	4.5	4.4	4.6	4.4
Non-Residential	0.6	0.3	2.0	2.0	3.4	3.5	4.4	4.5	4.3	4.4	4.2	4.3	4.3	4.4
Structures	1.5	0.4	3.0	2.6	5.0	4.5	6.5	5.8	6.3	5.7	6.1	5.5	6.2	5.6
Equipment	0.9	0.5	2.7	2.8	4.3	4.7	5.6	6.0	5.4	5.8	5.4	5.8	5.4	5.9
IP	0.3	0.2	1.3	1.2	2.1	2.0	2.5	2.5	2.4	2.4	2.3	2.3	2.3	2.3
Residential	3.4	0.7	4.6	4.0	7.1	6.8	9.8	9.1	9.4	8.8	9.1	8.6	9.2	8.7
Exports	0.7	0.5	2.6	2.2	4.4	3.6	5.5	4.3	5.4	4.3	5.3	4.2	5.3	4.2
Exports goods	0.9	0.6	3.2	2.6	5.3	4.3	6.6	5.2	6.5	5.1	6.4	5.0	6.5	5.0
Exports services	1.1	0.6	2.5	2.4	3.5	3.4	3.9	3.8	3.8	3.8	3.6	3.5	3.5	3.4
Imports	1.0	0.5	3.1	2.3	4.6	3.6	5.2	4.4	5.1	4.3	5.1	4.2	5.1	4.3
Imports goods	0.9	0.6	3.7	2.8	5.4	4.3	6.1	5.0	6.0	4.9	6.0	4.9	6.0	4.9
Imports services	2.4	0.5	2.3	1.8	2.5	2.7	3.1	3.2	3.0	3.2	2.9	3.0	2.9	3.0
Government	0.4	0.1	0.8	0.7	1.3	1.3	2.0	2.1	1.9	2.0	1.8	1.9	1.9	2.0
Federal	0.4	0.2	1.1	1.1	2.0	2.0	3.2	3.4	3.0	3.2	3.0	3.1	3.1	3.1
Defense	0.3	0.3	1.1	1.2	2.2	2.3	3.6	3.9	3.4	3.7	3.4	3.6	3.6	3.7
Non-Defense	2.1	0.6	2.9	2.1	3.6	3.1	4.3	3.7	4.2	3.7	4.2	3.6	4.3	3.7
State and local	0.5	0.1	0.7	0.5	0.9	0.9	1.3	1.2	1.2	1.2	1.1	1.1	1.0	1.1
Unemployment	0.1	0.1	0.4	0.4	0.6	0.6	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7
Employment	0.3	0.1	0.5	0.4	0.8	0.8	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Average Hours	0.2	0.0	0.3	0.2	0.4	0.3	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
Labor Force	0.2	0.0	0.2	0.2	0.3	0.3	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
Price Level	0.1	0.1	0.4	0.4	0.8	0.8	1.2	1.2	1.1	1.1	1.1	1.1	1.1	1.1
Nominal Wage	0.1	0.0	0.2	0.2	0.4	0.4	0.6	0.6	0.6	0.6	0.4	0.4	0.4	0.4
Real Wage	0.1	0.1	0.4	0.4	0.8	0.8	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1
Money Stock	0.2	0.2	1.0	1.0	1.7	1.7	2.5	2.5	2.4	2.4	2.2	2.2	2.2	2.2

The sample period is 1950:1-2019:2 except for Unemployment Rate and Labor Force (1951:1-2019:2), Nominal Interest Rate and Real Interest Rate (1957:3-2019:2), Nominal Monetary Base and Nominal Monetary Stock (1962:1-2019:2) and Average Hours, Nominal Wage and Real Wage (1967:1-2019:2).

Table 2.3 Correlation between Cyclical Component SU and SA Series by Filter

	Hamilton	HP	Band-Pass
GDP	0.912	0.608	0.942
Consumption	0.955	0.442	0.968
Goods	0.936	0.296	0.941
Durable goods	0.967	0.552	0.979
Nondurable goods	0.672	0.170	0.707
Services	0.694	0.621	0.662
Investment	0.972	0.924	0.989
Fixed investment	0.958	0.725	0.982
Non-Residential	0.949	0.846	0.976
Structures	0.925	0.782	0.972
Equipment	0.949	0.830	0.979
Intellectual Property	0.879	0.823	0.915
Residential	0.967	0.636	0.981
Exports	0.942	0.907	0.966
Exports goods	0.936	0.877	0.956
Exports services	0.942	0.780	0.942
Imports	0.626	0.726	0.749
Imports goods	0.650	0.762	0.763
Imports services	0.831	0.391	0.794
Government	0.923	0.852	0.961
Federal	0.946	0.938	0.974
Defense	0.967	0.980	0.981
Non-Defense	0.878	0.609	0.968
State and local	0.823	0.420	0.85
Unemployment Rate	0.990	0.904	0.999
Employment	0.979	0.724	0.997
Average Hours	0.943	0.627	0.984
Labor Force	0.513	0.506	0.989
Price Level	0.993	0.990	1.000
Nominal Wage	0.957	0.938	0.992
Real Wage	0.992	0.947	0.999
Nominal Money Stock	0.975	0.973	1.000

The sample period is 1950:1-2019:2 except for Unemployment Rate and Labor Force (1951:1-2019:2), Nominal Interest Rate and Real Interest Rate (1957:3-2019:2), Nominal Monetary Base and Nominal Monetary Stock (1962:1-2019:2) and Average Hours, Nominal Wage and Real Wage (1967:1-2019:2).

Table 2.4 Autocorrelations of the Cyclical Component by Filter

	Hamilton		HP		Band-Pass	
Order	Seasonally		Seasonally		Seasonally	
	Unadjusted	Adjusted	Unadjusted	Adjusted	Unadjusted	Adjusted
GDP						
1	0.904	0.899	0.064	0.819	0.907	0.896
2	0.771	0.768	0.354	0.584	0.678	0.654
3	0.618	0.624	-0.151	0.326	0.383	0.350
4	0.456	0.459	0.613	0.114	0.098	0.065
Consumption						
1	0.886	0.886	-0.262	0.803	0.924	0.918
2	0.781	0.779	0.231	0.636	0.721	0.702
3	0.642	0.637	-0.336	0.389	0.451	0.416
4	0.504	0.482	0.820	0.175	0.181	0.13
Investment						
1	0.882	0.895	0.652	0.788	0.892	0.890
2	0.755	0.769	0.474	0.549	0.642	0.636
3	0.607	0.622	0.223	0.291	0.335	0.327
4	0.427	0.430	0.164	0.050	0.056	0.048
Exports						
1	0.851	0.843	0.584	0.732	0.908	0.898
2	0.707	0.690	0.583	0.554	0.670	0.634
3	0.536	0.527	0.205	0.313	0.355	0.292
4	0.344	0.330	0.205	0.056	0.044	-0.036
Imports						
1	0.872	0.885	0.592	0.732	0.873	0.881
2	0.727	0.760	0.225	0.477	0.581	0.604
3	0.578	0.634	0.136	0.252	0.237	0.280
4	0.428	0.486	0.172	0.059	-0.044	0.014
Government						
1	0.910	0.899	0.551	0.904	0.919	0.927
2	0.795	0.775	0.585	0.733	0.706	0.732
3	0.673	0.647	0.224	0.522	0.429	0.472
4	0.525	0.476	0.366	0.296	0.157	0.211
Unemployment Rate						
1	0.928	0.918	0.673	0.900	0.918	0.917
2	0.801	0.796	0.525	0.693	0.698	0.697
3	0.661	0.647	0.313	0.450	0.409	0.408
4	0.511	0.490	0.364	0.222	0.126	0.125

	Hamilton		HP		Band-Pass	
Order	Seasonally		Seasonally		Seasonally	
	Unadjusted	Adjusted	Unadjusted	Adjusted	Unadjusted	Adjusted
Employment						
1	0.929	0.894	0.427	0.896	0.925	0.926
2	0.817	0.767	-0.026	0.712	0.726	0.727
3	0.675	0.647	0.232	0.503	0.457	0.457
4	0.525	0.487	0.618	0.284	0.179	0.179
Average Hours						
1	0.780	0.860	0.240	0.782	0.910	0.906
2	0.616	0.714	-0.383	0.527	0.673	0.663
3	0.473	0.554	0.092	0.276	0.370	0.354
4	0.311	0.368	0.637	0.069	0.080	0.063
Labor Force						
1	0.775	0.936	0.163	0.700	0.903	0.899
2	0.764	0.872	-0.578	0.447	0.664	0.647
3	0.591	0.813	0.077	0.296	0.366	0.330
4	0.619	0.792	0.776	0.185	0.093	0.040
Price Level						
1	0.854	0.849	0.890	0.909	0.930	0.930
2	0.695	0.684	0.709	0.748	0.765	0.765
3	0.528	0.520	0.552	0.568	0.541	0.542
4	0.347	0.336	0.380	0.368	0.302	0.303
Nominal Wage						
1	0.745	0.838	0.802	0.925	0.919	0.916
2	0.672	0.674	0.602	0.825	0.705	0.695
3	0.485	0.514	0.615	0.710	0.423	0.402
4	0.474	0.409	0.626	0.581	0.144	0.113
Real Wage						
1	0.890	0.879	0.784	0.893	0.944	0.945
2	0.766	0.743	0.516	0.724	0.791	0.793
3	0.629	0.622	0.473	0.544	0.573	0.575
4	0.481	0.469	0.402	0.329	0.327	0.328
Nominal Money Stock						
1	0.868	0.866	0.856	0.924	0.936	0.936
2	0.721	0.715	0.725	0.786	0.764	0.763
3	0.568	0.551	0.568	0.621	0.529	0.529
4	0.490	0.417	0.477	0.449	0.285	0.284

Table 2.5 Standard Deviation of the Cyclical Component by Filter

	Hamilton				HP				Band-Pass			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
GDP	2.66	2.59	2.51	2.56	1.92	1.14	1.11	1.15	0.94	0.97	0.93	0.99
Consumption	2.62	2.46	2.34	2.21	2.19	0.99	0.93	0.93	0.77	0.81	0.71	0.73
Durable	8.22	7.74	8.03	7.92	6.66	3.27	3.34	3.33	2.69	2.67	2.70	2.80
Nondurable	3.18	2.72	2.73	2.95	4.92	1.09	1.09	1.19	1.09	0.92	0.81	0.93
Services	1.73	2.73	1.84	1.64	0.87	0.71	0.63	0.61	0.50	0.57	0.48	0.46
Investment	12.31	11.18	11.95	12.21	7.88	6.92	6.96	7.08	5.88	5.89	6.04	6.16
Structures	9.56	9.70	8.83	9.23	9.64	8.50	4.87	8.44	8.50	8.63	4.89	8.64
Equipment	13.44	13.54	9.37	13.25	8.08	7.96	8.58	7.67	7.16	7.69	8.74	7.53
IP	3.39	3.33	13.32	3.24	2.72	1.35	7.60	1.49	1.33	1.11	7.45	1.48
Residential	17.53	12.93	3.34	12.29	13.63	9.12	1.65	8.91	4.89	5.53	1.48	5.34
Exports	4.96	4.91	4.85	4.83	3.68	3.43	3.37	3.31	3.34	3.46	3.38	3.41
Imports	7.94	7.66	7.86	7.58	5.13	4.35	4.27	4.11	3.90	4.12	4.03	3.95
Government	2.40	2.17	2.21	2.13	1.92	1.40	1.46	1.46	1.34	1.11	1.20	1.16
Defense	4.64	4.65	4.32	4.03	2.75	2.66	2.54	2.52	1.95	1.91	1.89	1.89
Non-defense	4.55	4.05	4.07	4.31	3.22	2.35	2.33	2.48	2.18	2.19	2.09	2.11
State and local	2.19	1.94	2.02	2.01	2.15	1.28	1.44	1.43	1.29	0.98	1.17	1.11
PGDP	-	0.84	0.88	1.55	-	0.42	0.45	0.45	-	0.43	0.47	0.47
PCON	-	0.89	0.92	0.95	-	0.58	0.60	0.61	-	0.58	0.60	0.62
PCORE	-	0.45	0.47	-	-	0.23	0.23	-	-	0.22	0.23	-

(1) Seasonally Unadjusted, (2): Pre-Revision Official, (3): Post-Revision Official, (4):Direct. The sample period is 2002Q1-2018Q1.